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Electronic transport in inhomogeneous quantum wires

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Abstract

We study the transport properties of a long non-uniform quantum wire where the electron–electron interactions and the density vary smoothly at large length scales. We show that these inhomogeneities lead to a finite resistivity of the wire, due to a weak violation of momentum conservation in the collisions between electrons. Estimating the rate of change of momentum associated with non-momentum-conserving scattering processes, we derive the expression for the resistivity of the wire in the regime of weakly interacting electrons and find a contribution linear in temperature for a broad range of temperatures below the Fermi energy. By estimating the energy dissipated throughout the wire by low-energy excitations, we then develop a different method for deriving the resistivity of the wire, which can be combined with the bosonization formalism. This allows us to compare our results with previous works relying on an extension of the Tomonaga–Luttinger model to inhomogeneous systems.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recent experiments on quantum wires and carbon nanotubes [1–15] have stimulated a lot of interest in the transport properties of one-dimensional conductors. From a theoretical point of view, interacting electrons in one dimension form the so-called Luttinger liquid [16, 17], whose properties qualitatively differ from the conventional Fermi liquid state. Recent progress in fabrication techniques has made possible the experimental observation of various characteristic signatures of the Luttinger liquid, such as the power-law behavior of the tunneling density of states [12–14], or the existence of separate spin and charge excitations [15]. It is also expected, within the Luttinger liquid theory, that the dc conductance of a quantum wire connected to Fermi liquid leads is given by the quantum of conductance $G_0 = 2e^2/h$ [18–20]. This quantization of the conductance has been reported in various experimental setups since its first observation in a quantum point contact [1, 2].

However, in a number of recent experiments [3–11], significant deviations from perfect quantization have been observed in the regime of low electron density. These deviations take the form of a shoulder-like structure below the first plateau of conductance. Although weak at the lowest temperatures available, this feature becomes more significant as the temperature increases, turning into a quasi-plateau at about $0.7 \times (2e^2/h)$. This so-called '0.7 structure', which is not expected in the Luttinger liquid theory, generated much theoretical interest, though there is at present no generally accepted microscopic theory. Most commonly, the experimental results are interpreted as originating from a spin-dependent mechanism. Such scenarios rely on a spontaneous spin polarization of the wire [3, 21, 22], or on the existence of a local spin-degenerate quasi-bound state whose screening would lead to Kondo-like effects [23, 24]. Other proposals considered various scattering mechanisms involving plasmons [25], spin waves [26] or phonons [27]. Several authors have also suggested that electron–electron interactions may affect the transport properties in quantum wire devices in a way that would be consistent with the '0.7 structure' [28–31].

In this context, a number of recent theory papers studied the electronic transport in a quantum wire modeled as a one-dimensional system in which the interactions are limited to a small region between two non-interacting leads. They concluded that the backscattering of either single electrons or pairs were the only mechanisms to significantly affect the transport properties of the system [30, 31], but only if the size of the interacting region is comparable to the Fermi wavelength of the electrons in the wire. If, on the other hand, the interaction strength varies smoothly over a much

larger distance, such backscattering processes only lead to exponentially small contributions which can be neglected. Using the model of a non-uniform Luttinger liquid with position-dependent parameters, it was found that no correction to the quantized conductance of the wire arises in this regime [18–20].

In this paper we show that even when the backscattering processes can be ignored, the non-uniformity of the interaction potential throughout the wire leads to a finite resistivity at non-zero temperatures. Indeed, the inhomogeneity of the interaction potential breaks the translational invariance of the system, allowing for two-particle scattering processes that conserve energy but not momentum. In section 2, we qualitatively show how some of these processes give rise to a finite resistivity and perform the corresponding calculation in section 3. In section 4, we present an alternative derivation of the resistivity in the language of the inhomogeneous Luttinger liquid model, allowing us to compare our results with previous works relying on this formalism [18–20]. Finally, in section 5 we discuss the relation of our results to the experiments probing the transport properties of inhomogeneous quantum wires. A brief summary of some of our results was reported in [32].

2. Qualitative picture

Let us consider an infinite one-dimensional system of weakly interacting electrons with a quadratic dispersion $\epsilon_p = p^2/2m$. To develop a qualitative picture of the physics involved, we restrict ourselves to the simple model of spinless electrons, with a uniform density n throughout the device. (We will tackle more realistic systems in the next section.) The inhomogeneity of the system comes from the electron–electron interaction whose strength varies smoothly along the wire.

When one enforces a dc current I to flow through the device, the electrons start moving and acquire a drift velocity v_d proportional to this applied current: $v_\mathrm{d} = I/ne$. In the reference frame moving with velocity v_d along the wire, the electronic subsystem is in an equilibrium state characterized by a Fermi energy ϵ_F and a temperature T. This was recently pointed out [33] in the context of Coulomb drag between two parallel wires.

As we are interested in the low-energy properties of the system, we focus on temperatures $T \ll \epsilon_\mathrm{F}$, so that the only relevant excitations are close to the Fermi level. As a result, one can isolate two well-defined branches corresponding to two species of fermions: the right- and left-moving electrons. Within each branch, the velocity of the electrons can be approximated by a constant and is given by $+v_\mathrm{F}$ and $-v_\mathrm{F}$ respectively for right- and left-movers. Upon changing from the moving to the stationary frame of reference, the electron velocities are modified in order to account for the drift velocity, and change from $\pm v_\mathrm{F}$ to $\pm v_\mathrm{F} + v_\mathrm{d}$. The consequences for the electron fluid as described in the stationary frame of reference are twofold. First, we need to introduce different Fermi energies for right- and left-moving electrons, $\epsilon_\mathrm{F} \to \epsilon_\mathrm{F}^{\mathrm{R,L}} = (1/2)m(v_\mathrm{F} \pm v_\mathrm{d})^2$. Second, since the density of states at the Fermi level is inversely proportional to velocity, we now have different densities of states for the two subsystems, $\nu \propto 1/v_\mathrm{F} \to \nu_{\mathrm{R,L}} \propto 1/(v_\mathrm{F} \pm v_\mathrm{d})$.

The latter result implies that the energy spacing between states is not only modified as we change the frame of reference, but also differs between the right and left branches in the stationary frame. Compared to the moving frame, the energy levels are stretched near the right Fermi point. This results in a somewhat broader distribution function, which can be interpreted as a slightly higher effective temperature T_R for the right-moving electrons (see figure 2). Similarly, near the left Fermi point, the energy levels are squeezed compared to the moving frame, resulting in a narrower distribution function, corresponding to a lower effective temperature T_L for the left-moving electrons. These effective temperatures follow the change in the density of states and are given by:

$$T_{\mathrm{R,L}} = T\left(1 \pm \frac{v_\mathrm{d}}{v_\mathrm{F}}\right) = T\left(1 \pm \frac{I}{env_\mathrm{F}}\right). \qquad (1)$$

The nature of these effective temperatures can be understood formally, by noticing that in the stationary frame, the system is no longer in thermal equilibrium because of the finite electric current. It follows that, quite generally, the occupation probability of a given state is no longer given by the standard Fermi–Dirac distribution. However, the introduction of the effective temperatures (1) for right- and left-movers enables one to write their occupation probabilities as Fermi functions of energy.

Because right- and left-movers have different temperatures, it is natural to expect that electron–electron interactions will give rise to thermalization between the two branches. In a uniform system, two-particle scattering processes cannot lead to thermalization as the conservation of both energy and momentum only allows processes which either exchange the momenta of the two incoming electrons or leave them unchanged [34]. On the other hand, in the case of inhomogeneous wires, the strength of the interaction potential is non-uniform so that the system is no longer translationally invariant, and two-particle scattering processes which conserve energy but not momentum are allowed.

A typical example of such electron–electron scattering processes is shown in figure 1. It describes the scattering of two electrons from an initial state with momenta p and k to a final state with momenta p' and k', and violates the momentum conservation: $p' + k' - p - k = P < 0$. Though the loss of momentum associated with this scattering process may affect the transport properties of the system, one could argue that it is compensated by an equal gain of momentum corresponding to the inverse process $(p', k') \to (p, k)$. This is however not the case here because of the temperature difference between the two branches: the processes involving a transfer of energy from the 'warmer' right-moving branch to the 'colder' left-moving one statistically occur more often than the corresponding inverse processes. As a result, the electronic system loses more momentum than it gains.

This overall loss of momentum can be viewed as resulting from a damping force, associated with the electron–electron collisions, and proportional to the temperature difference between the right- and left-moving branches. In order for a

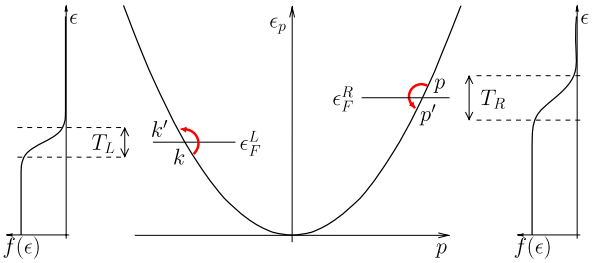

Figure 1. Electronic spectrum in the stationary frame, with the Fermi energies and effective temperatures for the right- and left-moving branches. The corresponding distribution functions near the right and left Fermi points are displayed as functions of energy. An example of non-momentum-conserving scattering process is provided.

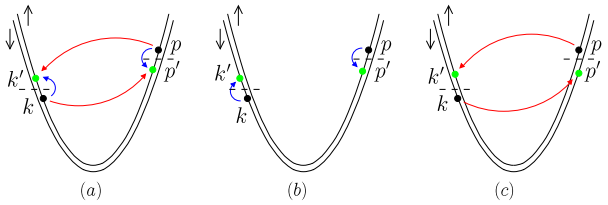

Figure 2. The three non-momentum-conserving processes that contribute to the resistivity. These processes can be designated using the standard notation involving coupling constants [35]: the scattering process represented in (a) corresponds to $g_{2\|} - g_{1\|}$, while (b) corresponds to $g_{2\perp}$, and (c) to $g_{1\perp}$. Following this correspondence, in the text we use the notation $\|$, $2\perp$ and $1\perp$ to refer respectively to (a), (b) and (c).

constant current to flow through the wire, this damping force has to be compensated by a driving force. The latter originates from a local electric field which appears as a response of the system to the external current. Using the force balance, and keeping in mind that the temperature difference $T_R - T_L \propto I$, this local electric field is proportional to the applied current bias. This implies a finite resistivity of the wire.

3. Weakly interacting electrons in the stationary frame

The above arguments provide a physical picture of how inhomogeneities lead to a finite resistivity. We now proceed with the calculation of the resistivity.

3.1. Model

Our starting point is a one-dimensional system of weakly interacting electrons with spins. In order to account for a non-uniform electron density $n(x)$, we introduce a one-particle potential $U(x)$ originating from the surrounding gates and impurities in the substrate. Moreover, the interaction between electrons is inhomogeneous, and described by a smoothly varying potential $V(r, R)$, given in the center-of-mass coordinates. The Hamiltonian for this system takes the form

$$H = H_0 + H_{\rm int} \tag{2a}$$

$$H_0 = \sum_{\gamma=\uparrow,\downarrow} \int \mathrm{d}x\ \psi_\gamma^\dagger(x) \left(-\frac{\hbar^2 \partial_x^2}{2m} + U(x) - \mu \right) \psi_\gamma(x) \tag{2b}$$

$$H_{\rm int} = \frac{1}{2} \sum_{\gamma,\beta} \int \mathrm{d}R \int \mathrm{d}r\ V(r, R)\ \psi_\gamma^\dagger \left(R + \frac{r}{2} \right) \psi_\beta^\dagger \left(R - \frac{r}{2} \right) \times \psi_\beta \left(R - \frac{r}{2} \right) \psi_\gamma \left(R + \frac{r}{2} \right), \tag{2c}$$

where $\psi_\gamma^\dagger(x)$ creates an electron with spin projection γ at position x, and μ is the chemical potential. We assume that the potential $U(x)$ is a smooth function of position, and that $U(x) \ll \mu$. This allows us to introduce a position-dependent Fermi energy $\epsilon_{\rm F}(x) = \mu - U(x)$. Similarly, the position-dependent Fermi momentum and velocity are straightforwardly defined as $p_{\rm F}(x) = \sqrt{2m\epsilon_{\rm F}(x)}$ and $v_{\rm F}(x) = p_{\rm F}(x)/m$.

We keep a very general form for the interaction potential between electrons and only make the following assumptions concerning its characteristic length scales. On the one hand, we assume for simplicity that the interaction is short range, the potential decaying rapidly as a function of the distance r between electrons. On the other hand, since we consider a non-uniform system, the interaction depends on the position R of the center of mass. The variations with respect to R are smooth and occur at a typical length scale d, large compared to both the Fermi wavelength and the range of the interaction potential. Similarly, we assume that the potential $U(x)$ varies at the same typical length scale d as the interaction strength.

3.2. Resistivity

We focus now on temperatures in a broad range $\hbar v_{\rm F}/d \ll T \ll \epsilon_{\rm F}$. In order to compute the resistivity of the wire, we consider a force balance on a small isolated segment of wire taken at position x, whose length Δx well exceeds the range of the interaction while satisfying $\hbar v_{\rm F}/T \ll \Delta x \ll d$. When an external current I is applied to the device, the response of the system manifests itself as a local electric field $E(x) = \rho(x)I$, which in turn leads to a driving force $eE(x)n(x)\Delta x$ acting on the electrons. This driving force is compensated by a damping force ΔF resulting from the inhomogeneous electron–electron interaction, so that the resistivity can be written as

$$\rho(x) = -\frac{\Delta F}{en(x)I\Delta x}. \tag{3}$$

The damping force can be evaluated as the change in momentum per unit time associated with two-particle scattering processes. In the regime $k_{\rm F}d \gg 1$, the processes with a large momentum difference compared with the Fermi momentum lead to exponentially small contributions. As a result, in what follows we focus on processes which only weakly violate the momentum conservation (see figure 2).

Because of the non-uniformity of the wire, strictly speaking the momentum of the electron is not a well-defined quantity. However, since $U(x)$ varies smoothly over a length scale $d \gg \Delta x$, it is possible to introduce a well-defined momentum over the size of the small segment under

consideration. The expression for the momentum thus depends on the position x of the small segment, and for a state of energy ϵ is given by $p_\epsilon(x) = \pm\sqrt{2m[\epsilon - U(x)]}$. Here the + sign corresponds to the right branch, the − sign to the left one.

Similarly, the eigenstates of the free Hamiltonian are no longer given by simple plane waves but instead satisfy $[-\hbar^2\partial_x^2/2m + U(x)]\Psi_\epsilon(x) = \epsilon\Psi_\epsilon(x)$. Keeping in mind that the typical length scale d associated with the inhomogeneities of the wire is much larger than the Fermi wavelength, we use the semiclassical approximation, which yields

$$\Psi_{\epsilon,\pm}(x) = \frac{1}{\sqrt{\hbar|v_\epsilon(x)|}} \exp\left\{\pm\frac{\mathrm{i}}{\hbar}\int_0^x \mathrm{d}x'\sqrt{2m\,[\epsilon - U(x')]}\right\} \tag{4}$$

normalized according to $\int \mathrm{d}x\,\Psi_{\epsilon,\pm}(x)\Psi^*_{\epsilon',\pm}(x) = 2\pi\delta(\epsilon - \epsilon')$. Here the velocity is defined as $v_\epsilon(x) = p_\epsilon(x)/m$ and the index $\pm$ refers to the right/left branches. We ignored the backscattered wave, since it only leads to exponentially small contributions for $k_\mathrm{F}d \gg 1$.

The rate of change of momentum associated with the three processes shown in figure 2 is evaluated using the Fermi golden rule, so that the damping force acting on the electrons takes the form

$$\begin{aligned}\Delta F = \frac{2\pi}{\hbar}\sum_{p,k,p',k'}&\left(\left|V^{\parallel}_{pk;p'k'}\right|^2 + \left|V^{2\perp}_{pk;p'k'}\right|^2 + \left|V^{1\perp}_{pk;p'k'}\right|^2\right)\\ &\times \delta(\epsilon_p + \epsilon_k - \epsilon_{p'} - \epsilon_{k'})\left(p' + k' - p - k\right)\\ &\times [f^\mathrm{R}_p f^\mathrm{L}_k(1 - f^\mathrm{R}_{p'})(1 - f^\mathrm{L}_{k'}) - f^\mathrm{R}_{p'} f^\mathrm{L}_{k'}(1 - f^\mathrm{R}_p)\\ &\times (1 - f^\mathrm{L}_k)],\end{aligned} \tag{5}$$

where we introduced $V_{pk;p'k'}$ as the matrix element of the interacting Hamiltonian (2c) for scattering from the initial state (p, k) to the final state (p', k') according to the processes shown in figure 2. The superscripts $\parallel$, 2 $\perp$ and 1 $\perp$ refers to the standard notation for these scattering processes [35]. The occupation numbers $f^{\mathrm{R,L}}$ introduced in (5) are given by the Fermi distribution evaluated with the appropriate temperatures $T_{\mathrm{R,L}}(x)$, defined in section 2.

One readily sees from (5) that the damping force vanishes at $T_\mathrm{R} = T_\mathrm{L}$. Using the fact that the temperature difference $T_\mathrm{R} - T_\mathrm{L} \propto I$ is small in the linear response regime, we expand the occupation numbers $f^{\mathrm{R,L}}$ to first order in $T_\mathrm{R} - T_\mathrm{L}$. To avoid redundant derivations, let us focus on the first process shown on figure 2(a). The damping force corresponding to this scattering process is then given by

$$\begin{aligned}\Delta F_\parallel = -\frac{I}{32\pi^2 e\epsilon_\mathrm{F}(x)}&\int \mathrm{d}\epsilon_p\,\mathrm{d}\epsilon_k\,\mathrm{d}\epsilon_{p'}\,\mathrm{d}\epsilon_{k'}\\ &\times \left|V^{\parallel}\left(\epsilon_p, \epsilon_k; \epsilon_{p'}, \epsilon_{k'}\right)\right|^2 \frac{\epsilon_{p'} - \epsilon_p + \epsilon_k - \epsilon_{k'}}{T}\\ &\times \left(p' + k' - p - k\right)\delta(\epsilon_p + \epsilon_k - \epsilon_{p'} - \epsilon_{k'})\\ &\times f^\mathrm{R}_p f^\mathrm{L}_k(1 - f^\mathrm{R}_{p'})(1 - f^\mathrm{L}_{k'}),\end{aligned} \tag{6}$$

where we converted the summations over states into energy integrals, and introduced the matrix element $V^{\parallel}(\epsilon_p, \epsilon_k; \epsilon_{p'}, \epsilon_{k'})$ evaluated using the set of eigenstates defined in (4).

Note that the expansion in $T_\mathrm{R} - T_\mathrm{L}$ leads to an expression for the damping force which is proportional to the applied current. As a result, in the linear response regime we can ignore any further dependence on I, as this would lead to contributions that are non-linear in the current bias. This allows us to use the Fermi energy ϵ_F and velocity v_F as they are defined in the equilibrium state, i.e. in the reference frame where the electric current vanishes.

Focusing on states close to the Fermi energy, we can simplify the expression for the eigenstates (4) of the free Hamiltonian into

$$\Psi_{\epsilon,\pm}(x) \simeq \Psi_{\epsilon_\mathrm{F},\pm}(x)\exp\left[\pm\mathrm{i}(\epsilon - \epsilon_\mathrm{F})\int_0^x \frac{\mathrm{d}x'}{\hbar v_\mathrm{F}(x')}\right], \tag{7}$$

where $\Psi_{\epsilon_\mathrm{F},\pm}(x)$ is obtained from (4) by setting $\epsilon = \epsilon_\mathrm{F}$. This allows us to estimate the matrix element $V^{\parallel}$ to first order in the interaction:

$$\begin{aligned}V^{\parallel}\left(\epsilon_p, \epsilon_k; \epsilon_{p'}, \epsilon_{k'}\right) = &\int_x^{x+\Delta x}\mathrm{d}R\,\frac{1}{[\hbar v_\mathrm{F}(R)]^2}\\ &\times \exp\left(\mathrm{i}\int_0^R \mathrm{d}x'\frac{\epsilon_{p'} - \epsilon_p + \epsilon_k - \epsilon_{k'}}{\hbar v_\mathrm{F}(x')}\right)\\ &\times \int_{-\Delta x}^{\Delta x}\mathrm{d}r\;V(r, R)\left(1 - \mathrm{e}^{-2\mathrm{i}k_\mathrm{F}(R)r}\right).\end{aligned} \tag{8}$$

Here we introduced the Fermi wavevector $k_\mathrm{F}(R) = p_\mathrm{F}(R)/\hbar$.

Using the fact that p and p' on the one hand, and, k and k' on the other hand, are on the same branch, we express the momentum difference in terms of a difference in energy by introducing the density of states. We then define $\varepsilon = \epsilon_{p'} - \epsilon_p + \epsilon_k - \epsilon_{k'}$, and perform the remaining energy integrals. Combining the resulting expression for the damping force with (3), and substituting the matrix element (8), we obtain the following expression for the resistivity associated with the scattering process of figure 2(a)

$$\begin{aligned}\rho_\parallel(x) = &\frac{T}{64e^2\epsilon_\mathrm{F}(x)v_\mathrm{F}(x)n(x)\Delta x}\\ &\times \int_x^{x+\Delta x}\mathrm{d}R_1\frac{V_0(R_1) - V_{2k_\mathrm{F}}(R_1)}{\pi\hbar v_\mathrm{F}(R_1)}\\ &\times \int_x^{x+\Delta x}\mathrm{d}R_2\frac{V_0(R_2) - V_{2k_\mathrm{F}}(R_2)}{\pi\hbar v_\mathrm{F}(R_2)}\\ &\times \int \mathrm{d}\varepsilon\left[\frac{\varepsilon/4T}{\sinh(\varepsilon/4T)}\right]^2\frac{\varepsilon^2}{\hbar v_\mathrm{F}(R_1)\hbar v_\mathrm{F}(R_2)}\\ &\times \exp\left(\mathrm{i}\varepsilon\int_{R_1}^{R_2}\frac{\mathrm{d}x'}{\hbar v_\mathrm{F}(x')}\right).\end{aligned} \tag{9}$$

The shortened forms V_0 and V_{2k_F} correspond to the zero-momentum and $2k_\mathrm{F}$ Fourier components of the potential $V(r, R)$ with respect to its first variable r defined as

$$\begin{aligned}V_0(R) &= \int \mathrm{d}r\;V(r, R) \qquad \text{and}\\ V_{2k_\mathrm{F}}(R) &= \int \mathrm{d}r\;V(r, R)\mathrm{e}^{\mathrm{i}2k_\mathrm{F}(R)r}.\end{aligned} \tag{10}$$

At this stage, it is convenient to rewrite the energy integral in (9) by replacing ε^2 with a second derivative of the exponential term with respect to R_1 and R_2, along with the appropriate factors of $\hbar v_\mathrm{F}$. Performing an integration by parts in the position variables leaves us with an expression

involving single derivatives of the dimensionless parameters $V_0(R)/[\pi\hbar v_F(R)]$ and $V_{2k_F}(R)/[\pi\hbar v_F(R)]$. The remaining integral over ε can be easily simplified by noticing that it is the Fourier transform of a rapidly decaying function which only extends over a range of energy comparable to temperature. For temperatures $T \gg \hbar v_F/d$, it reduces to a delta function in $R_1 - R_2$ which allows us to simplify (9) to

$$\rho_\parallel(x) = \frac{h}{64e^2}\frac{T}{n(x)\epsilon_F(x)}\left[\partial_x\left(\frac{V_0(x) - V_{2k_F}(x)}{\pi\hbar v_F(x)}\right)\right]^2, \quad (11)$$

where we expanded the remaining position integral to first order in Δx.

The contributions corresponding to the remaining two scattering processes can be computed following the same steps and are readily obtained from (11) by replacing $V_0(x) - V_{2k_F}(x)$ with $V_0(x)$ for $\rho_{2\perp}(x)$, and with $V_{2k_F}(x)$ for $\rho_{1\perp}(x)$. Combining the contributions from all three processes, the final expression for the resistivity in the regime of temperatures $T \gg \hbar v_F/d$ takes the form

$$\rho(x) = \frac{h}{64e^2}\frac{T}{n(x)\epsilon_F(x)}\left\{\left[\partial_x\left(\frac{V_0(x) - V_{2k_F}(x)}{\pi\hbar v_F(x)}\right)\right]^2 + \left[\partial_x\left(\frac{V_0(x)}{\pi\hbar v_F(x)}\right)\right]^2 + \left[\partial_x\left(\frac{V_{2k_F}(x)}{\pi\hbar v_F(x)}\right)\right]^2\right\}. \quad (12)$$

This expression clearly stresses that the meaningful inhomogeneous quantity is not just the interaction potential but rather the dimensionless parameter that involves both the electron–electron interaction and the Fermi velocity. In particular, this means that a system with a non-uniform density but homogeneous interactions between electrons still displays a non-zero resistivity.

4. Weakly interacting electrons in the moving frame

We now introduce a different approach for evaluating the resistivity of the system. Unlike the derivation of the previous section, this new treatment is compatible with the bosonization formalism. Along with providing an alternative derivation of the result (12), our goal in developing this approach is to compare with the results of previous works on the inhomogeneous Tomonaga–Luttinger liquid [18–20].

4.1. Bosonization

Previous attempts at studying the transport properties of quantum wires relied on an extension of the Tomonaga–Luttinger model to inhomogeneous systems [18–20]. These authors assumed that the inhomogeneities do not change the form of the Hamiltonian, and can be accounted for by introducing position-dependent velocities and Luttinger liquid parameters. In the general case, however, a rigorous derivation of the bosonized Hamiltonian for these systems is still lacking. Here we show how such a bosonized Hamiltonian can be derived explicitly in the case of a non-uniform system of weakly interacting electrons.

The standard bosonization formula for weakly interacting fermions involves the Fermi momentum as well as a momentum cutoff (see e.g. [17]), and as such cannot be straightforwardly extended to non-uniform systems where both these quantities can develop a position dependence. The key idea then is to map the inhomogeneous system of electrons onto a set of fictitious fermions described by a Hamiltonian whose non-interacting part is translationally invariant. From there, a standard bosonization procedure holds and the resulting Hamiltonian expressed in terms of the new variables is very reminiscent of the conjectured inhomogeneous Luttinger liquid Hamiltonian, in the limit of weak interactions (see section 5).

Our starting point is similar to the one we considered in section 3, namely a system of interacting electrons with a non-uniform density described by the Hamiltonian (2). As we noticed in the previous section, the non-uniform potential $U(x)$ appearing in (2*b*) breaks the translational invariance of the system, already when no electron–electron interaction is present. As a result, the eigenstates of the free Hamiltonian are no longer plane waves but for energies close to the Fermi level, they can be approximated by (7).

Up to a prefactor which depends on position but not on ϵ, these low-energy eigenstates look like plane waves, putting forth a more natural set of variables: the energy difference $\epsilon - \epsilon_F$ and $X(x) = \int_0^x dx'/\hbar v_F(x')$. An expansion of the electron field operator $\psi_{\gamma,\pm}(x)$ over these plane waves calls for the introduction of a fictitious fermion field operator $\eta_{\gamma,\pm}(X)$ defined as

$$\psi_{\gamma,\pm}(x) = \Psi_{\epsilon_F,\pm}(x)\,\eta_{\gamma,\pm}(X(x)), \quad (13)$$

where $\Psi_{\epsilon_F,\pm}(x)$ was introduced in (7). Note that the anti-commutation relations satisfied by $\psi_{\gamma,\pm}(x)$ transfer to $\eta_{\gamma,\pm}(X)$ ensuring that $\{\eta_{\gamma,\sigma}(X), \eta^\dagger_{\beta,\sigma'}(Y)\} = \delta_{\sigma\sigma'}\delta_{\gamma\beta}\delta(X - Y)$.

Let us now derive the Hamiltonian describing the physics of these fictitious fermions. This is accomplished by substituting (13) into the Hamiltonian (2). By construction, the free Hamiltonian is translationally invariant in the new variable X. At low energy, the interacting part of the Hamiltonian can be decomposed in three sectors corresponding to the conventional g_1, g_2 and g_4 processes [35]. The main difference here is that the associated coupling constants are now position dependent. They can be obtained from the Fourier components of the electron–electron interaction potential.

As an example, consider the so-called $g_{2\parallel}$ process. Following [17], the coupling constant for this process is given by the zero-momentum Fourier component of the interaction potential, which in the case of our inhomogeneous system corresponds to $V_0(x)$, introduced in (10). Replacing ψ with η according to (13), and introducing the density operator $\nu_{\gamma,\pm}(X) = \eta^\dagger_{\gamma,\pm}(X)\eta_{\gamma,\pm}(X)$, the $g_{2\parallel}$ process retains the same form

$$\int dx\; g_{2\parallel}(x)\rho_{\gamma,\sigma}(x)\rho_{\gamma,-\sigma}(x) \longrightarrow \pi\int dX\; y_{2\parallel}(X)\nu_{\gamma,\sigma}(X)\nu_{\gamma,-\sigma}(X) \quad (14)$$

only with a dimensionless coupling constant given by $y_{2\parallel}(X(x)) = g_{2\parallel}(x)/\pi\hbar v_F(x)$. A similar treatment can be applied to the remaining sectors of the interaction.

The resulting Hamiltonian expressed in terms of the fictitious field η can now be bosonized following the standard procedure:

$$\eta_{\uparrow,\pm}(X) = \frac{U_{\gamma,\pm}}{\sqrt{2\pi\alpha}} \exp\left\{ \frac{-\mathrm{i}}{\sqrt{2}}[\pm\phi_\rho(X) - \theta_\rho(X) \pm \phi_\sigma(X) - \theta_\sigma(X)] \right\} \tag{15a}$$

$$\eta_{\downarrow,\pm}(X) = \frac{U_{\gamma,\pm}}{\sqrt{2\pi\alpha}} \exp\left\{ \frac{-\mathrm{i}}{\sqrt{2}}[\pm\phi_\rho(X) - \theta_\rho(X) \mp \phi_\sigma(X) + \theta_\sigma(X)] \right\} \tag{15b}$$

where we introduced the fields ϕ_ν and θ_ν (with $\nu = \rho, \sigma$) satisfying bosonic commutation relations $[\phi_\nu(X), \partial_Y \theta_\nu(Y)] = \mathrm{i}\pi\delta(X-Y)$. Here $U_{\gamma,\pm}$ are the standard Klein factors [17] and α^{-1} is an energy cutoff[1] introduced to regularize the theory in the ultraviolet sector.

In terms of the bosonic variables, the Hamiltonian of the system can be written as a sum of two terms describing the excitations of charge and spin degrees of freedom respectively, and takes the form

$$H = H_\rho + H_\sigma \tag{16a}$$

$$H_\rho = \frac{1}{2\pi}\int \mathrm{d}X \left[\left(\partial_X \theta_\rho\right)^2 + \left(1 + y_\rho(X)\right)\left(\partial_X \phi_\rho\right)^2 \right] \tag{16b}$$

$$H_\sigma = \frac{1}{2\pi}\int \mathrm{d}X \left[(\partial_X \theta_\sigma)^2 + (1 - y_\sigma(X))\,(\partial_X \phi_\sigma)^2 \right] + \frac{2}{(2\pi\alpha)^2}\int \mathrm{d}X\; y_\sigma(X)\cos\left(2\sqrt{2}\phi_\sigma\right). \tag{16c}$$

The dimensionless parameters y_ρ and y_σ are conventional notation for combinations of y_1, y_2 and y_4 [17] given by

$$y_\rho\,(X(x)) = \frac{V_{2k_\mathrm{F}}(x) - 2V_0(x)}{\pi\hbar v_\mathrm{F}(x)} \qquad y_\sigma\,(X(x)) = \frac{V_{2k_\mathrm{F}}(x)}{\pi\hbar v_\mathrm{F}(x)}, \tag{17}$$

where V_0 and V_{2k_F} are the Fourier components of the interaction potential as defined in (10).

Note that this form of bosonization clearly highlights that the important variables are the dimensionless parameters $y_{\rho,\sigma}$ rather than the various interaction constants $g_{1,2,4}$. The calculation of the resistivity carried out in section 3 led to the same observation, see (12).

4.2. Resistance and dissipation

A way to determine the resistivity of the system is to relate it to the mechanism of dissipation of energy into the wire when an external current bias is applied. This relation was explored in [36] in the context of a quantum wire in the Wigner crystal regime, and the method we outline here is similar.

In the presence of an applied current $I = I_0 \cos\omega t$, the electrons start moving in the wire. More specifically, in the dc limit $\omega \to 0$, one can assume that the current is uniform throughout the wire and all electrons move in phase. As a result, the position of the electrons depends on time and is related to the injected charge $q(t) = I_0\omega^{-1}\sin\omega t$ defined as $I(t) = \dot{q}(t)$. This time dependence of the positions of the electrons can be accounted for by replacing $x \to x + q(t)/en(x)$ in the position-dependent parameters of the Hamiltonian. While this has no effect in practice when the translational invariance holds, for an inhomogeneous system it leads to a time-dependent perturbation to the Hamiltonian. Alternatively, this amounts to describing the system in the reference frame moving with the electron fluid. In this case, the electrons experience the effect of an inhomogeneous potential moving as a function of time.

In terms of the fictitious set of fermions $\eta_{\gamma,\pm}(X)$, one needs to substitute X in the dimensionless interaction parameters $y_{\rho,\sigma}$ by the time-dependent position $X + q(t)/en(X)$ where the density in these variables is given by $n(X(x)) = n(x)\hbar v_\mathrm{F}(x)$. In the linear response regime, an expansion to first order in $q(t)$ leads to the following form of the Hamiltonian:

$$H \longrightarrow H + \int \mathrm{d}X \frac{q(t)}{en(X)}\mathcal{H}'(X), \tag{18}$$

where we introduced the notation $\mathcal{H}'(X) = \mathcal{H}'_\rho(X) + \mathcal{H}'_\sigma(X)$ for the following quantities:

$$\mathcal{H}'_\rho(X) = \frac{1}{2\pi}\left[\partial_X y_\rho(X)\right]\left(\partial_X\phi_\rho\right)^2 \tag{19a}$$

$$\mathcal{H}'_\sigma(X) = -\frac{1}{2\pi}\left[\partial_X y_\sigma(X)\right](\partial_X\phi_\sigma)^2 + \frac{2}{(2\pi\alpha)^2}\left[\partial_X y_\sigma(X)\right]\cos(2\sqrt{2}\phi_\sigma). \tag{19b}$$

The time-dependent perturbation in (18) acts as an external driving force, resulting in the creation of spin and charge excitations. These excitations are responsible for dissipating the energy from the external force into the wire. Using the Fermi golden rule, it is possible to estimate the rates of these absorption and emission processes, and therefore, the energy W dissipated in unit time into the system. In the linear response regime, where the amplitude I_0 of the current oscillations is weak, the energy dissipated in unit time is quadratic in I_0 and is given by

$$W = \hbar\omega\frac{2\pi}{\hbar}\left(\frac{I_0}{2e\omega}\right)^2\int_{-\infty}^{+\infty}\frac{\mathrm{d}t}{2\pi\hbar}\left(\mathrm{e}^{\mathrm{i}\omega t} - \mathrm{e}^{-\mathrm{i}\omega t}\right) \times \int\frac{\mathrm{d}X}{n(X)}\int\frac{\mathrm{d}Y}{n(Y)}\langle\mathcal{H}'(X,t)\mathcal{H}'(Y,0)\rangle, \tag{20}$$

where $\langle\cdots\rangle$ stands for thermodynamic averaging.

The resistance of the system is then derived by comparing the dissipated energy obtained in (20) with the Joule heat law $W = I_0^2 R/2$. Since the charge part (19*a*) and the spin part (19*b*) of the time-dependent perturbation commute, one expects the resistance R of the wire to be expressed as the sum of a spin and a charge contribution $R = R_\rho + R_\sigma$, which can be evaluated separately. This can be understood as the consequence of having two independent channels for dissipating energy throughout the wire, corresponding to spin and charge excitations [36]. After some manipulations, these

[1] Considering that the fictitious fermions were introduced in the vicinity of the electron Fermi surface, one should assume $\alpha^{-1} \ll \epsilon_\mathrm{F}$.

two contributions to the resistance can be expressed in the dc limit as

$$R_\nu = -\frac{1}{\hbar e^2}\int \frac{\mathrm{d}X}{n(X)}\int \frac{\mathrm{d}Y}{n(Y)}\lim_{\omega\to 0}\frac{\mathrm{Im}\left[\mathcal{W}_{\mathrm{ret},\nu}(X,Y;\omega)\right]}{\omega}. \tag{21}$$

Here we introduced the retarded correlator $\mathcal{W}_{\mathrm{ret},\nu}$ as the Fourier transform in time of $\mathcal{W}_{\mathrm{ret},\nu}(X,Y;t) = -\mathrm{i}\theta(t)\langle[\mathcal{H}'_\nu(X,t),\mathcal{H}'_\nu(Y,0)]\rangle$, where $\nu = \rho,\sigma$.

4.3. Charge contribution to the resistivity

In order to derive the contribution to the resistivity from charge degrees of freedom, we substitute in (21) the expression for $\mathcal{H}'_\rho$ introduced in (19*a*). The retarded correlator $\mathcal{W}_{\mathrm{ret},\rho}$ resulting from this substitution is quartic in the bosonic field ϕ_ρ. It is thus more convenient to express it in terms of the corresponding time-ordered correlation function, via an analytic continuation in frequency space. This allows for the use of Wick's theorem, ultimately leading to the following expression of the retarded correlator:

$$\mathcal{W}_{\mathrm{ret},\rho}(X,Y;\omega) = -\frac{\hbar}{2\pi^2}\left[\partial_X y_\rho(X)\right]\left[\partial_Y y_\rho(Y)\right] \times\left[\int_0^\beta \mathrm{d}\tau \mathrm{e}^{\mathrm{i}\nu_n\tau}\left(\frac{\partial^2}{\partial X\partial Y}\langle T\phi_\rho(X,\tau)\phi_\rho(Y,0)\rangle\right)^2\right]_{i\nu_n\to\hbar\omega+i\delta}, \tag{22}$$

where $\langle T\ldots\rangle$ corresponds to the time-ordered correlation function.

Since (22) is explicitly quadratic in the interaction, the dominant contribution to the retarded correlator $\mathcal{W}_{\mathrm{ret},\rho}$ can be derived by using the free propagator of the bosonic field ϕ_ρ. The latter is readily obtained from the non-interacting Hamiltonian, and is given by

$$\langle T\phi_\rho(X,\tau)\phi_\rho(Y,0)\rangle = T\sum_n\int\frac{\mathrm{d}K}{2\pi}\frac{\pi}{\nu_n^2+K^2}\mathrm{e}^{\mathrm{i}K(X-Y)}\mathrm{e}^{-\mathrm{i}\nu_n\tau}. \tag{23}$$

Combining (23) with (22), performing the integral over imaginary time τ, and substituting the analytically continued result into (21), the charge contribution to the resistance in the dc limit reads:

$$R_\rho = -\frac{\pi\hbar T^2}{8e^2}\int \mathrm{d}X\frac{\partial_X y_\rho(X)}{n(X)}\int\mathrm{d}Y\frac{\partial_Y y_\rho(Y)}{n(Y)} \times\frac{1}{\sinh^2\left[2\pi T(X-Y)\right]}\left\{1-\frac{2\pi T(X-Y)}{\tanh\left[2\pi T(X-Y)\right]}\right\}, \tag{24}$$

where we restricted ourselves to contributions up to second order in the interaction.

One recognizes in (24) a rapidly decaying integral kernel for $|X-Y|\gg 1/T$. In terms of real space quantities, this corresponds to distances of order $\hbar v_\mathrm{F}/T$. It follows that at temperatures $T\gg\hbar v_\mathrm{F}/d$ the double integral in X and Y is dominated by short-range contributions. This allows us to reduce the expression (24) for the resistance to a single integral over X. Changing variables back from X to x, the integrand of the resulting expression for the resistance R_ρ can be identified with the charge contribution to the resistivity at position x in space and is given by

$$\rho_\rho(x) = \frac{h}{128e^2}\frac{T}{\epsilon_\mathrm{F}(x)n(x)}\left[\partial_x y_\rho(x)\right]^2, \tag{25}$$

where we focused on temperatures in the range $\hbar v_\mathrm{F}/d\ll T\ll\epsilon_\mathrm{F}$.

4.4. Spin contribution to the resistivity

The method used above to derive the charge contribution to the resistivity can be readily extended to evaluate that of spin degrees of freedom. Substituting the expression for $\mathcal{H}'_\sigma$ into (21), one notices that the spin contribution splits off into two parts: one coming from the quadratic term in ϕ_σ, the other from the cosine term.

4.4.1. Quadratic term. The expressions for the quadratic parts of $\mathcal{H}'_\rho$ and $\mathcal{H}'_\sigma$ are identical up to a sign, upon replacing the charge parameter y_ρ and field ϕ_ρ by their spin counterparts. As a result, in order to derive the contribution to the transport properties from the quadratic term in ϕ_σ, it is sufficient to repeat the steps leading to (25) but replace y_ρ by y_σ so that

$$\rho_{\sigma,\mathrm{quad}}(x) = \frac{h}{128e^2}\frac{T}{\epsilon_\mathrm{F}(x)n(x)}\left[\partial_x y_\sigma(x)\right]^2. \tag{26}$$

Here we again restricted ourselves to temperatures in the range $\hbar v_\mathrm{F}/d\ll T\ll\epsilon_\mathrm{F}$.

4.4.2. Cosine term. The contribution to the resistivity coming from the cosine term of the spin Hamiltonian can be inferred from (26) based on the following symmetry argument. In terms of the bosonized Hamiltonian, the interaction-dependent term appearing in the quadratic part of H_σ accounts for the coupling between z components of the electron spins [17]. This term ultimately leads to the contribution $\rho_{\sigma,\mathrm{quad}}$ obtained in (26). On the other hand, the cosine term in (16*c*) corresponds to the coupling of the remaining x and y components [17]. Because of the SU(2) symmetry, the contributions from all three components of the interaction between electron spins are the same. As a result, we expect the cosine term to contribute twice as much to the resistivity as the quadratic part of the spin Hamiltonian.

This can be verified explicitly by substituting the cosine term from (19*a*) into the expression for the retarded correlator $\mathcal{W}_{\mathrm{ret},\sigma}$. After performing the analytic continuation and taking the dc limit $\omega\to 0$, the cosine–cosine correlation function takes the form

$$\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\left\{\left[\int_0^\beta \mathrm{d}\tau\mathrm{e}^{\mathrm{i}\nu_n\tau}\left\langle T\cos\left(2\sqrt{2}\phi_\sigma(X,\tau)\right)\right.\right.\right. \times\left.\left.\left.\cos\left(2\sqrt{2}\phi_\sigma(Y,0)\right)\right\rangle\right]_{\mathrm{i}\nu_n\to\hbar\omega+\mathrm{i}\delta}\right\} = -\frac{\pi^3\hbar T^2\alpha^4}{\sinh^2\left[2\pi T(X-Y)\right]}\left\{1-\frac{2\pi T(X-Y)}{\tanh\left[2\pi T(X-Y)\right]}\right\}. \tag{27}$$

Here it is sufficient to perform the thermodynamic averaging using the free Hamiltonian since this term enters $\mathcal{W}_{\mathrm{ret},\sigma}$ with a prefactor quadratic in the interaction parameter y_σ.

One recognizes in (27) the same short-range kernel we encountered in (24). It follows that, at temperatures $T \gg \hbar v_\mathrm{F}/d$, one can simplify the expression for the resistance into a single integral over X. Changing back variables from X to x, and identifying the integrand in x with the resistivity, we obtain the contribution from the cosine term $\rho_{\sigma,\cos}(x) = 2\rho_{\sigma,\mathrm{quad}}(x)$, as we argued from the SU(2) symmetry. The total contribution from spin degrees of freedom thus amounts to three times the result (26).

Combining the charge and spin contributions, the final expression for the resistivity of the wire at temperatures $T \gg \hbar v_\mathrm{F}/d$ is given by

$$\rho(x) = \frac{h}{128e^2}\frac{T}{\epsilon_\mathrm{F}(x)n(x)}\left\{\left[\partial_x y_\rho(x)\right]^2 + 3\left[\partial_x y_\sigma(x)\right]^2\right\}. \tag{28}$$

Using (17) to replace the dimensionless parameters $y_\rho(x)$ and $y_\sigma(x)$ with their expression in terms of the electron–electron interaction potential, the latter result becomes identical to (12).

5. Discussion

5.1. Inhomogeneous Luttinger liquid

It is interesting to compare the bosonized Hamiltonian we derived in (16) to that of the inhomogeneous Tomonaga–Luttinger model conjectured in [18–20]. To do so, we change variables back from X to x, so that the Hamiltonian (16) takes the form

$$H = H_\rho + H_\sigma \tag{29a}$$

$$H_\rho = \int \mathrm{d}x\, \frac{\hbar v_\mathrm{F}(x)}{2\pi}\left[\left(\partial_x\vartheta_\rho\right)^2 + \left(1+y_\rho(x)\right)\left(\partial_x\varphi_\rho\right)^2\right] \tag{29b}$$

$$H_\sigma = \int \mathrm{d}x\, \frac{\hbar v_\mathrm{F}(x)}{2\pi}\left[(\partial_x\vartheta_\sigma)^2 + (1-y_\sigma(x))\,(\partial_x\varphi_\sigma)^2\right] + \int \mathrm{d}x\, \frac{2g_\sigma(x)}{[2\pi\alpha\hbar v_\mathrm{F}(x)]^2}\cos\left(2\sqrt{2}\varphi_\sigma\right), \tag{29c}$$

where we denoted $\varphi_\nu(x) = \phi_\nu(X(x))$ and $\vartheta_\nu(x) = \theta_\nu(X(x))$ (with $\nu = \rho, \sigma$).

The charge Hamiltonian H_ρ and the quadratic part of H_σ are identical to the inhomogeneous Tomonaga–Luttinger Hamiltonian [18–20], taken in the limit of weak interactions. The important difference comes from the cosine term of the spin Hamiltonian. This term was absent from previous works which either discarded it arguing that the coupling constant g_σ renormalizes towards zero at low energy scales [37], or simply focused on a system of spinless fermions [18–20]. To recover a standard form for the cosine term, one needs to introduce a position-dependent momentum cutoff $[\alpha(x)]^{-1}$ defined as $\alpha(x) = \alpha\hbar v_\mathrm{F}(x)$, where α^{-1} is the energy cutoff introduced in (15). Keeping in mind that momentum is no longer a conserved quantity in our model, this position dependence of the momentum cutoff was to be expected. Interestingly though, the natural guess relying on the common interpretation of $\alpha(x)$ as a small distance cutoff, would have led to a different answer. Indeed, assuming that $\alpha(x)$ represents the shortest inter-particle distance, one would expect it to be inversely proportional to the electron density, i.e. $\alpha(x) \propto 1/v_\mathrm{F}(x)$.

Although the quadratic part of our bosonized Hamiltonian is similar to the model considered [18–20], we do not reach the same final answer. This is because our treatment amounts to considering perturbations to the Luttinger liquid Hamiltonian which were not taken into account in previous studies. In section 4, we treated $I(t)$ as an external parameter. In the framework of the Luttinger liquid model, it can also be interpreted as an excitation of the charge mode. Using the bosonization expression for the electric current $I = e(\sqrt{2}/\pi)\dot{\phi_\rho}$, one readily sees that $q(t)$ can thus appear as a dynamical variable, directly proportional to the charge field ϕ_ρ. As a result, the linear in $q(t)$ perturbation to the Hamiltonian in (18) corresponds, in the conventional Luttinger liquid theory, to cubic terms in the bosonic fields of the form $\phi_\rho(\partial_X\phi_\nu)^2$ ($\nu = \rho, \sigma$). These terms are irrelevant perturbations to the Luttinger liquid Hamiltonian, and as such are usually discarded. However, it was proven that within the quadratic Luttinger liquid Hamiltonian, non-uniform electron–electron interactions do not contribute to the resistance [18–20]. It thus makes sense to take these irrelevant perturbations into account. Our approach showed that they affect the transport properties in a non-trivial way and lead to a finite resistivity.

5.2. Connection with experiments

Our results are relevant to experiments performed on long quantum wires. However, we focused on the case of weakly interacting electrons which is unlikely to be realized in experimental situations. Let us discuss to what extent our conclusions are modified when this restriction on the interaction strength is relaxed.

Though our results are not readily applicable in the case of strong electron–electron interactions, the method developed in section 4 which relies on bosonization suggests that the temperature and density dependences of the resistivity should not be affected by the strength of the interactions. Experimental measurements of these dependences may thus be compared with our results.

Furthermore, given the Hamiltonian of the system in the strongly interacting regime, one could repeat the treatment of section 4 in order to derive the resistivity. Unfortunately, a rigorous derivation of the bosonized Hamiltonian in the case of a strongly interacting inhomogeneous system is yet to be found.

5.3. Equilibration

In our derivation, we assumed that the electronic subsystem is in equilibrium in the moving frame. For this to be satisfied, we need the wires to be longer than the typical length scale l_eq associated with the processes of equilibration taking place inside the wire. If the size of the wire becomes too short with respect to the equilibration length l_eq, we expect our results to be modified by an additional small prefactor of the order of the ratio of these two length scales. This might lead to a non-trivial temperature dependence of the resistance, depending on the leading equilibration mechanism involved.

Little is known about equilibration mechanisms in one-dimensional interacting systems. In the case of weakly interacting electrons, recent work [34] suggests that scattering processes involving three electrons may be the leading source of equilibration in the system. Because of consideration of energy and momentum conservation, these three-particle collisions should involve states near the bottom of the band, resulting in a strong suppression at low temperatures. In the experimentally relevant case of low electron density and strong interactions, this analysis no longer holds and a detailed treatment remains elusive. It is natural to expect that the equilibration in the wire would become easier as the interactions grow stronger.

6. Summary

In this paper we studied the effect of inhomogeneous electron–electron interactions on the transport properties of a quantum wire. We considered a very general form of the interaction potential, and allowed for a non-uniform density of electrons along the wire. We argued that the inhomogeneities allow for non-momentum-conserving scattering processes which give rise to a finite resistivity of the wire. We showed that in the regime of weakly interacting electrons, such scattering processes contribute to the resistivity as a linear in T term[2], over a broad range of temperatures T below the Fermi energy. We also reformulated our results within the framework of the inhomogeneous Tomonaga–Luttinger model, and analyzed the differences with previous works relying on this formalism.

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